Notes

qxtr01 <qxtr01@gmail.com>

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1 Symbols

1.1 Greek alphabet

A	α	alpha
B	β	beta
Γ	γ	gamma
Δ	δ	delta
E	ϵ	epsilon
Z	ζ	zeta
H	η	eta
Θ	θ	theta
I	ι	iota
K	κ	kappa
Λ	λ	lambda
M	μ	mu
N	ν	nu
Ξ	ξ	xi
O	o	omicron
Π	π	pi
P	ρ	rho
Σ	σ	sigma
T	τ	tau
Υ	υ	upsilon
Φ	ϕ	phi
X	χ	chi
Ψ	ψ	psi
Ω	ω	omega

2 Logic

2.1 Zeroth-order logic

P	$\bot$	$\top$
$\bot$	$\bot$	$\top$
$\top$	$\bot$	$\top$

P	P	$\neg P$
$\bot$	$\bot$	$\top$
$\top$	$\top$	$\bot$

P	Q	$\bot$	$\top$
$\bot$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\bot$	$\top$
$\top$	$\bot$	$\bot$	$\top$
$\top$	$\top$	$\bot$	$\top$

P	Q	$P \wedge Q$	$P \uparrow Q$
$\bot$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\bot$	$\top$
$\top$	$\bot$	$\bot$	$\top$
$\top$	$\top$	$\top$	$\bot$

P	Q	$P \not\rightarrow Q$	$P \rightarrow Q$
$\bot$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\bot$	$\top$
$\top$	$\bot$	$\top$	$\bot$
$\top$	$\top$	$\bot$	$\top$

P	Q	P	$\neg P$
$\bot$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\bot$	$\top$
$\top$	$\bot$	$\top$	$\bot$
$\top$	$\top$	$\top$	$\bot$

P	Q	$P \not\leftarrow Q$	$P \leftarrow Q$
$\bot$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\top$	$\bot$
$\top$	$\bot$	$\bot$	$\top$
$\top$	$\top$	$\bot$	$\top$

P	Q	Q	$\neg Q$
$\bot$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\top$	$\bot$
$\top$	$\bot$	$\bot$	$\top$
$\top$	$\top$	$\top$	$\bot$

P	Q	$P \not\leftrightarrow Q$	$P \leftrightarrow Q$
$\bot$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\top$	$\bot$
$\top$	$\bot$	$\top$	$\bot$
$\top$	$\top$	$\bot$	$\top$

P	Q	$P \vee Q$	$P \downarrow Q$
$\bot$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\top$	$\bot$
$\top$	$\bot$	$\top$	$\bot$
$\top$	$\top$	$\top$	$\bot$

$$
\begin{aligned}
P \lor P &\iff P \\
P \lor Q &\iff Q \lor P \\
(P \lor Q) \lor R &\iff P \lor (Q \lor R)
\end{aligned}
$$

$$
\begin{aligned}
P \land P &\iff P \\
P \land Q &\iff Q \land P \\
(P \land Q) \land R &\iff P \land (Q \land R)
\end{aligned}
$$

$$
P \lor (Q \land R) = (P \lor Q) \land (P \lor R)
$$
$$
P \land (Q \lor R) = (P \land Q) \lor (P \land R)
$$

$$
\begin{aligned}
\neg(P \lor Q) &\iff \neg P \land \neg Q \\
\neg(P \land Q) &\iff \neg P \lor \neg Q
\end{aligned}
$$

2.2 First-order logic

$$
\begin{aligned}
\forall x(P(x)) &\iff \neg\exists x(\neg P(x)) \\
\exists x(P(x)) &\iff \neg\forall x(\neg P(x)) \\
\forall x(\forall y(P(x,y))) &\iff \forall y(\forall x(P(x,y))) \\
\exists x(\exists y(P(x,y))) &\iff \exists y(\exists x(P(x,y))) \\
\forall x(P(x)) \land \forall x(Q(x)) &\iff \forall x(P(x) \land Q(x)) \\
\exists x(P(x)) \lor \exists x(Q(x)) &\iff \exists x(P(x) \lor Q(x))
\end{aligned}
$$

$$
\begin{aligned}
\forall P(x)(Q(x)) &\iff \forall x(P(x) \to Q(x)) \\
\exists P(x)(Q(x)) &\iff \exists x(P(x) \land Q(x))
\end{aligned}
$$

3 Set theory

3.1 Sets

1. axiom of extensionality: $\forall a(\forall b(a = b \leftrightarrow \forall c(c \in a \leftrightarrow c \in b)))$

$$
\begin{aligned}
a = b &\iff \forall c(c \in a \leftrightarrow c \in b) \\
a \subseteq b &\iff \forall c(c \in a \rightarrow c \in b) \\
a = b &\iff a \subseteq b \wedge b \subseteq a
\end{aligned}
$$

2. axiom of pair: $\forall a(\forall b(\exists c(\forall d(d \in c \leftrightarrow d = a \vee d = b))))$
3. axiom of union: $\forall a(\exists b(\forall c(c \in b \leftrightarrow \exists d(d \in a \wedge c \in d))))$
4. axiom of power set: $\forall a(\exists b(\forall c(c \in b \leftrightarrow c \subseteq a)))$
5. axiom schema of comprehension: $\forall a(\exists b(\forall c(c \in b \leftrightarrow c \in a \wedge P(c))))$

$$
\begin{aligned}
\bigcup a = \{x \in u : \exists b(b \in a \wedge x \in b)\} \\
\bigcap a = \{x \in u : \forall b(b \in a \wedge x \in b)\}
\end{aligned}
$$

$$
\begin{aligned}
a' &= \{x \in u : x \notin a\} \\
a \cup b &= \{x \in u : x \in a \vee x \in b\} \\
a \cap b &= \{x \in u : x \in a \wedge x \in b\} \\
a \setminus b &= \{x \in u : x \in a \wedge x \notin b\}
\end{aligned}
$$

$$
\begin{aligned}
a \cup a &= a \\
a \cup b &= b \cup a \\
(a \cup b) \cup c &= a \cup (b \cup c)
\end{aligned}
$$

$$
\begin{aligned}
a \cap a &= a \\
a \cap b &= b \cap a \\
(a \cap b) \cap c &= a \cap (b \cap c)
\end{aligned}
$$

$$
\begin{aligned}
a \cup (b \cap c) = (a \cup b) \cap (a \cup c) \\
a \cap (b \cup c) = (a \cap b) \cup (a \cap c)
\end{aligned}
$$

$$
\begin{aligned}
(a \cup b)' = a' \cap b' \\
(a \cap b)' = a' \cup b'
\end{aligned}
$$

$$P_a = \{x \in u : x \subseteq a\}$$

6. axiom of empty set: $\exists a(\forall b(b \notin a))$
7. axiom of infinity: $\exists a(\emptyset \in a \wedge \forall b(b \in a \rightarrow b \cup \{b\} \in a))$

- P partition of A:
  - $\forall p \in P(p \neq \emptyset)$
  - $\forall p_1 \in P(\forall p_2 \in P(p_1 \neq p_2 \rightarrow p_1 \cap p_2 = \emptyset))$
  - $\bigcup P = A$

3.2 Tuples

$$\langle a, b\rangle = \{\{a\}, \{a, b\}\}$$

$$A \otimes B = \{x \in P_{P_{(A \cup B)}} : \exists a(\exists b(a \in A \wedge b \in B \wedge x = \langle a, b\rangle))\}$$

$$N_n = \{m \in N \setminus \{0\} : m \in n\}$$

$$\prod_{i=1}^{n} A_i = \{x \in P_{N_n \otimes \bigcup F} : \forall i \in N_n : \exists! a_i \in A_i : \langle i, a_i\rangle \in x\}$$

$$
\begin{aligned}
() = \{\} \\
(a_1, \ldots, a_n) = \bigcup_{i \in N_n} \{\langle i, a_i\rangle\}
\end{aligned}
$$

3.3 Binary relations

$$R = (A, B, G) \qquad G \subseteq A \times B$$

$$\operatorname{dom} R = \{a \in A : \exists b \in B((a, b) \in G)\}$$
$$\operatorname{codom} R = \{b \in B : \exists a \in A((a, b) \in G)\}$$

$$R[A] = \{b \in \operatorname{codom} R : \exists a \in A((a, b) \in R)\}$$

- total: $\forall a \in A : \exists b \in B : (a, b) \in G$
- surjective: $\forall b \in B : \exists a \in A : (a, b) \in G$
- functional: $\forall a \in A : \forall b_1, b_2 \in B : (a, b_1) \in G \land (a, b_2) \in G \to b_1 = b_2$
- injective: $\forall b \in B : \forall a_1, a_2 \in A : (a_1, b) \in G \land (a_2, b) \in G \to a_1 = a_2$
- function: total, functional
- bijective function: total, functional, injective, surjective

$$R = (A, B, G) \qquad G \subseteq A \times B$$
$$R^{-1} = (B, A, G') \qquad G' \subseteq B \times A$$
$$G' = \{c \in B \times A : \exists a \exists b (c = (a, b) \land (b, a) \in G)\}$$

$$R = (A, B, G_R)$$
$$S = (B, C, G_S)$$
$$S \circ R = (A, C, G_{S \circ R})$$

$$G_{S \circ R} = \{d \in A \times C : \exists a \exists c (d = (a, c) \land \exists b \in B((a, b) \in R \land (b, c) \in S))\}$$

3.4 Binary relations over a set

$$R = (A, A, G) \qquad G \subseteq A \times A$$

- reflexive: $\forall a \in A : (a, a) \in G$
- symmetric: $\forall a_1, a_2 \in A : (a_1, a_2) \in G \to (a_2, a_1) \in G$
- antisymmetric: $\forall a_1, a_2 \in A : (a_1, a_2) \in G \land (a_2, a_1) \in G \to a_1 = a_2$
- transitive: $\forall a_1, a_2, a_3 \in A : (a_1, a_2) \in G \land (a_2, a_3) \in G \to (a_1, a_3) \in G$
- equivalence relation: reflexive, symmetric, transitive
- partial order: reflexive, antisymmetric, transitive

3.5 Equivalence relations

- $\sim$ equivalence relation on A. $a \in A$.
  - $[a]_\sim = \{x \in A : x \sim a\} = \{x \in A : a \sim x\}$
- $\sim$ equivalence relation on A.
  - $A_{/\sim} = \{x \in P_A : \exists a \in A(x = [a]_\sim)\}$

3.6 Partial orders

- $\preceq$ partial order of A. $B \subseteq A$
  - $b \in B$ least element of B: $\forall x \in B(b \preceq x)$
  - $b \in B$ greatest element of B: $\forall x \in B(x \preceq b)$
  - $a \in A$ lower bound of B: $\forall x \in B(a \preceq x)$
  - $a \in A$ upper bound of B: $\forall x \in B(x \preceq a)$

3.7 Functions

$$\begin{aligned} f &: A \to B \\ a &\mapsto f(a) \end{aligned}$$

3.8 Cardinality

$$\begin{gathered} |A| \in \mathbb{N} \\ |P_A| = 2^{|A|} \end{gathered}$$

$$\begin{gathered} |A|, |B| \in \mathbb{N} \\ |B^A| = |B|^{|A|} \end{gathered}$$

4 Algebra

4.1 Natural numbers

$$n \in \mathbb{N}$$
$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

$$n \in \mathbb{N}$$
$$n! = \begin{cases} 1 & n = 0 \\ n(n-1)! & n \geq 1 \end{cases}$$

$$n, k \in \mathbb{N} \qquad k \leq n$$
$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n}{0} = \binom{n}{n} = 1$$
$$\binom{n}{1} = \binom{n}{n-1} = n$$

$$n, k \in \mathbb{N} \setminus \{0\} \qquad k < n$$
$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

4.2 Integer numbers

$$a \in \mathbb{Z} \setminus \{0\} \qquad b \in \mathbb{Z}$$
$$a \mid b \iff \exists h \in \mathbb{Z} : b = ha$$

$$a, b \in \mathbb{Z} \qquad n \in \mathbb{N} \setminus \{0\}$$
$$a \equiv b \pmod{n} \iff n \mid (b - a)$$

$$[x]_n = \{y \in \mathbb{Z} : x \equiv y \pmod{n}\}$$

$$\mathbb{Z}_n = \{[0]_n, \dots, [n-1]_n\}$$

$$a \equiv b \pmod{n} \qquad c \equiv d \pmod{n}$$
$$a + c \equiv b + d \pmod{n}$$
$$ac \equiv bd \pmod{n}$$

$$\gcd : \mathbb{Z}^2 \setminus \{(0,0)\} \to \mathbb{N} \setminus \{0\}$$
$$(z_1, z_2) \mapsto \gcd(z_1, z_2)$$

$$\operatorname{lcm} : \mathbb{Z} \setminus \{0\} \times \mathbb{Z} \setminus \{0\} \to \mathbb{N} \setminus \{0\}$$
$$(z_1, z_2) \mapsto \operatorname{lcm}(z_1, z_2)$$

4.3 Real numbers

$$\sqrt{2} = 1.4142135623\dots$$
$$\sqrt{3} = 1.7320508075\dots$$
$$\sqrt{5} = 2.2360679774\dots$$
$$e = 2.7182818284\dots$$
$$\pi = 3.1415926535\dots$$

4.4 Binary operations

$$* : A \times A \to A$$
$$(a_1, a_2) \mapsto a_1 * a_2$$

- $*$ associative: $\forall a_1, a_2, a_3 \in A : (a_1 * a_2) * a_3 = a_1 * (a_2 * a_3)$
- $*$ commutative: $\forall a_1, a_2 \in A : a_1 * a_2 = a_2 * a_1$

- $u_l \in A$ left identity: $\forall a \in A : u_l * a = a$
- $u_r \in A$ right identity: $\forall a \in A : a * u_r = a$
- $u \in A$ identity: $\forall a \in A : u * a = a * u = a$
- $\bar{a}_l \in A$ left inverse of $a \in A$: $\bar{a}_l * a = u$
- $\bar{a}_r \in A$ right inverse of $a \in A$: $a * \bar{a}_r = u$
- $\bar{a} \in A$ inverse of $a \in A$: $\bar{a} * a = a * \bar{a} = u$

4.5 Algebraic structures

- $(A, *)$ semigroup:
  1. $\forall a_1, a_2, a_3 \in A : (a_1 * a_2) * a_3 = a_1 * (a_2 * a_3)$
- $(A, *)$ monoid:
  1. $\forall a_1, a_2, a_3 \in A : (a_1 * a_2) * a_3 = a_1 * (a_2 * a_3)$
  2. $\exists u \in A : \forall a \in A : u * a = a * u = a$
- $(A, *)$ group:
  1. $\forall a_1, a_2, a_3 \in A : (a_1 * a_2) * a_3 = a_1 * (a_2 * a_3)$
  2. $\exists u \in A : \forall a \in A : u * a = a * u = a$
  3. $\forall a \in A : \exists \bar{a} \in A : \bar{a} * a = a * \bar{a} = u$
- $(A, +, \cdot)$ ring:
  - $(A, +)$ Abelian group
    1. $\forall a_1, a_2, a_3 \in A : (a_1 + a_2) + a_3 = a_1 + (a_2 + a_3)$
    2. $\exists 0 \in A : \forall a \in A : 0 + a = a + 0 = a$
    3. $\forall a \in A : \exists (-a) \in A : (-a) + a = a + (-a) = 0$
    4. $\forall a_1, a_2 \in A : a_1 + a_2 = a_2 + a_1$
  - $(R, \cdot)$ monoid
    1. $\forall a_1, a_2, a_3 \in A : (a_1 \cdot a_2) \cdot a_3 = a_1 \cdot (a_2 \cdot a_3)$
    2. $\exists 1 \in A : \forall a \in A : 1 \cdot a = a \cdot 1 = a$
  - $\forall a_1, a_2, a_3 \in A : (a_1 + a_2) \cdot a_3 = (a_1 \cdot a_3) + (a_2 \cdot a_3)$
  - $\forall a_1, a_2, a_3 \in A : a_1 \cdot (a_2 + a_3) = (a_1 \cdot a_2) + (a_1 \cdot a_3)$
- $(A, +, \cdot)$ field:
  - $(A, +)$ Abelian group
    1. $\forall a_1, a_2, a_3 \in A : (a_1 + a_2) + a_3 = a_1 + (a_2 + a_3)$
    2. $\exists 0 \in A : \forall a \in A : 0 + a = a + 0 = a$
    3. $\forall a \in A : \exists (-a) \in A : (-a) + a = a + (-a) = 0$
    4. $\forall a_1, a_2 \in A : a_1 + a_2 = a_2 + a_1$
  - $(R, \cdot)$ Abelian monoid

1. $\forall a_1, a_2, a_3 \in A : (a_1 \cdot a_2) \cdot a_3 = a_1 \cdot (a_2 \cdot a_3)$
2. $\exists 1 \in A : \forall a \in A : 1 \cdot a = a \cdot 1 = a$
3. $\forall a_1, a_2 \in A : a_1 \cdot a_2 = a_2 \cdot a_1$

- $\forall a_1, a_2, a_3 \in A : (a_1 + a_2) \cdot a_3 = (a_1 \cdot a_3) + (a_2 \cdot a_3)$
- $\forall a_1, a_2, a_3 \in A : a_1 \cdot (a_2 + a_3) = (a_1 \cdot a_2) + (a_1 \cdot a_3)$
- $\forall a \neq 0 : \exists a^{-1} : a \cdot a^{-1} = a^{-1} \cdot a = 1$

4.6 Vector spaces

- $(U, +, \cdot)$ vector space over K:
  - $(U, +)$ abelian group
  - $\forall a \in K(\forall \mathbf{u}, \mathbf{v} \in U(a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}))$
  - $\forall a, b \in K(\forall \mathbf{u} \in U((a + b)\mathbf{u} = a\mathbf{u} + b\mathbf{u}))$
  - $\forall a, b \in K(\forall \mathbf{u} \in U(a(b\mathbf{u}) = (ab)\mathbf{u}))$
  - $\forall \mathbf{u} \in U(1\mathbf{u} = \mathbf{u})$

4.7 Radicals

$$a \in \{x \in \mathbb{R} : x \geq 0\} \qquad n \in \{m \in \mathbb{N} : 2 \mid m\}$$
$$\sqrt[n]{a}$$

$$a \in \mathbb{R} \qquad n \in \{m \in \mathbb{N} : \neg(2 \mid m)\}$$
$$\sqrt[n]{a}$$

4.8 Exponentiation

$$a \in \mathbb{R} \qquad n \in \mathbb{N}$$
$$a^n = \begin{cases} 1 & n = 0 \\ a \cdot a^{n-1} & n \geq 1 \end{cases}$$

$$a \in \mathbb{R} \setminus \{0\} \qquad n \in \mathbb{N}$$
$$a^{-(n+1)} = \frac{1}{a^{n+1}}$$

4.9 Positional numeral systems

$$b \in \mathbb{N} \setminus \{0,1\}$$
$$a_0, \ldots, a_m \in \mathbb{R}$$
$$c_1, \ldots, c_n \in \mathbb{R}$$
$$(a_m \cdots a_0 . c_1 \cdots c_n)_b = \sum_{i=0}^{m} a_i b^i + \sum_{i=1}^{n} c_i b^{-i}$$

4.10 Polynomials

$$p : \mathbb{R} \to \mathbb{R}$$
$$p(x) = \sum_{i=0}^{n} a_i x^i \qquad n \in \mathbb{N}$$

$$\deg : \mathbb{R}[x] \setminus \{0_{\mathbb{R}[x]}\} \to \mathbb{N}$$

$$n \in \mathbb{N}$$
$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$ax^2 + bx + c = 0 \qquad a \neq 0$$
$$b^2 - 4ac \geq 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

5 Mathematical analysis

5.1 Real numbers

$$x_0 \in \mathbb{R} \qquad \delta \in \{x \in \mathbb{R} : x > 0\}$$
$$I_\delta(x_0) = (x_0 - \delta, x_0 + \delta)$$
$$I_\delta^-(x_0) = (x_0 - \delta, x_0]$$
$$I_\delta^+(x_0) = [x_0, x_0 + \delta)$$

$$\delta \in \mathbb{R}$$
$$I_\delta(+\infty) = (\delta, +\infty)$$
$$I_\delta(-\infty) = (-\infty, \delta)$$

5.2 Functions of a real variable

$$\operatorname{dom} f \subseteq \mathbb{R}$$
$$f : \operatorname{dom} f \to \mathbb{R}$$

- increasing: $\forall x_1, x_2 \in \operatorname{dom} f (x1 < x2 \to f(x_1) \leq f(x_2))$
- strictly increasing: $\forall x_1, x_2 \in \operatorname{dom} f (x1 < x2 \to f(x_1) < f(x_2))$
- decreasing: $\forall x_1, x_2 \in \operatorname{dom} f (x1 < x2 \to f(x_1) \geq f(x_2))$
- strictly decreasing: $\forall x_1, x_2 \in \operatorname{dom} f (x1 < x2 \to f(x_1) > f(x_2))$

5.3 Trigonometric functions

$$(\sin(x))^2 + (\cos(x))^2 = 1$$

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = 2\cos^2(x) - 1$$

5.4 Other functions

$$\operatorname{abs} : \mathbb{R} \to \mathbb{R}$$
$$\operatorname{abs}(x) = \begin{cases} x & x > 0 \\ 0 & x = 0 \\ -x & x < 0 \end{cases}$$

$$\text{sgn} : \mathbb{R} \to \{-1, 0, 1\}$$

$$\text{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

5.5 Limits

$$x_0 \in \bar{\mathbb{R}} \qquad A \subseteq \mathbb{R}$$

$$x_0 \in D(A) \iff \forall I_\delta(x_0) \in I(x_0) : A \cap I_\delta(x_0) \setminus \{x_0\} \neq \emptyset$$

$$f : D_f \subseteq \mathbb{R} \to \mathbb{R} \qquad x_0 \in D(D_f) \qquad l \in \bar{\mathbb{R}}$$

$$\lim_{x \to x_0} f(x) = l$$

$$\forall I_\epsilon(l) \in I(l) : \exists I_\delta(x_0) \in I(x_0) : \forall x \in I_\delta(x_0) \setminus \{x_0\} : f(x) \in I_\epsilon(l)$$

$$f : D_f \subseteq \mathbb{R} \to \mathbb{R} \qquad x_0 \in D(D_f \cap I_{x_0}(\pm\infty)) \qquad l \in \bar{\mathbb{R}}$$

$$\lim_{x \to x_0^\pm} f(x) = l$$

$$\forall I_\epsilon(l) \in I(l) : \exists I_\delta^\pm(x_0) \in I^\pm(x_0) : \forall x \in I_\delta^\pm(x_0) \setminus \{x_0\} : f(x) \in I_\epsilon(l)$$

6 Physics

6.1 Prefixes

10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deca	da
10^{0}		
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

7 Computer science

7.1 Prefixes

2^{60}	exbi	Ei
2^{50}	pebi	Pi
2^{40}	tebi	Ti
2^{30}	gibi	Gi
2^{20}	mebi	Mi
2^{10}	kibi	Ki